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**FORECASTING THE NAVAL OFFICER PERSONNEL
STRUCTURE TO ESTIMATE BASIC PAY**

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6 FORECASTING THE NAVAL OFFICER PERSONNEL
FORCE STRUCTURE TO ESTIMATE BASIC PAY

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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A primary concern of Navy management is the ability to meet the Navy's manpower requirements in terms of both the quantity and quality of personnel. Recruitment, promotion, retirement, and other managerial policies are all directly related to and restricted by the Military Personnel, Navy (MPN) budget. Because about 17 percent of the MPN budget is allocated for officer basic pay (approximately \$1 billion), special attention must be given to the accurate forecasting of officer | | |

populations. The Naval Pay Predictor, Officer (NAPPO) Model, designed to accomplish this objective, relies solely upon historical population data and user-supplied end-strengths. Time series analysis is used to determine a general set of forecasting models that adequately explain the historical data. Other statistical procedures, including those employed in determining the cost of paying the officer force and in estimating newly commissioned officer input populations, are also described. Validation results indicating errors of less than .21 percent, .64 percent, and 1.06 percent for 1-, 2-, and 3-year lead times, respectively, for total officer basic pay are presented. These results represent an improvement of more than 80 percent over the average of the last 9 years of forecasting by the budget planners.

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FOREWORD

The effort described in this report supports the development of compensation and cost models, an exploratory development objective under Task Area ZF55.521.010. The objective of this task area is to develop techniques to forecast the cost and behavior effects of the Navy's compensation policies. The main effort in FY79 was directed toward solving two problems. The first concerns the development of quantitative methods for evaluating alternative retirement systems. The objective is to forecast the cost and retention behavior effects of such systems on the enlisted force. The second problem is forecasting the Navy's military manpower budget to avoid cost overruns and personnel turbulence. This problem has been addressed by developing the Naval Personnel Pay Predictor, Officer (NAPPO) model, which is described in this report, and the companion Naval Personnel Pay Predictor, Enlisted (NAPPE) model for enlisted personnel, which is currently being used by the Deputy Chief of Naval Operations (MP&T) to forecast enlisted basic pay and to distribute the enlisted force over 31 length-of-service categories. The NAPPO model will be combined with the NAPPE model and other forecasting techniques to yield a Military Personnel, Navy (MPN) account forecasting model, to be used by OP-131, the Financial Management Branch. Liaison will be maintained with OP-131 as to when the all-MPN forecasting model will be installed.

This report is the second in a series documenting work in the area of budget forecasting. The first, NPRDC Technical Report 78-4, described the development of the NAPPE model.

DONALD F. PARKER
Commanding Officer

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SUMMARY

Problem

Navy personnel planning and policy decisions often involve problems of predicting future events or conditions. One such problem is that of forecasting the Navy's military manpower budget to avoid cost overruns and personnel policy turbulence. In particular, cost overruns can occur due to inaccurate predictions of either the personnel force structure (personnel arrayed by length-of-service (LOS) and pay grade) or the various pay rates. To control the first source of error, forecasting models are being developed to predict required obligations based on a desired or allocated man-year average by pay grade.

Objective

The objective of this effort was to provide a detailed description of the analysis and procedures used to formulate a model for forecasting Navy officer force structures and basic pay obligations. The model, which will be used to monitor the service age characteristics of the force and its costs in basic pay, is known as the Naval Personnel Pay Predictor, Officer (NAPPO).

Approach

NAPPO relies upon historical USN, USNR, USNT, and All Navy (ALNAV) quarterly force structure files dating back to 1963. Time series analysis techniques were applied to these files to find a particular set of time series models that would be appropriate for forecasting the LOS marginal distribution of each array. Various combinations of these forecasts were then compared to obtain a "best" forecast for the ALNAV LOS distribution. Additional statistical procedures, previously developed for the Naval Personnel Pay Predictor, Enlisted (NAPPE) model, were used for (1) deriving the interior of the force structure matrix given the forecasted LOS and inputted pay grade marginal distributions, (2) forecasting the force structure for personnel with less than 1 year of service or more than 30 years, (3) costing the force structure, (4) estimating average strength, and (5) validating the model.

Findings

The statistical techniques employed in NAPPO proved to be highly accurate in producing estimates of officer basic pay. Validation results indicated forecasting errors of less than .3 percent for FY76, FY77, and FY78. NAPPO's predictions for mean LOS of the force also indicated a high degree of accuracy. As expected, the accuracy of the forecasts generally diminished as the forecast lead time increased.

Conclusions

1. Sufficient data are available to allow time series analysis techniques to be applied toward the development of a model to forecast the officer force structure and, hence, basic pay obligations.
2. A particular set of time series models explain a great majority of the data. This set of models provides a solid statistical basis for making force structure forecasts.
3. Comparisons show that the NAPPO model is much more accurate than the current procedures used to forecast officer basic pay.

Recommendation

The NAPPO model should be used by Navy management to provide future basic pay and mean LOS estimates.

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INTRODUCTION

Problem

The Navy's ability to meet its manpower requirements, in terms of both quantity and quality of personnel, is restricted by the size of the personnel budget. Recruitment, promotion, retirement, and other managerial policies are all directly or indirectly related to the amount of dollars available in the Military Personnel, Navy (MPN) budget, of which 17 percent, or approximately 1 billion dollars, is allocated for basic pay of officer personnel. In fiscal year 1978, \$1,093,603,000 of obligations were incurred for this one item. Thus, even a 1 percent discrepancy in forecasting the officer basic pay component may result in an over- or underexpenditure of \$10 million. Because the amount of basic pay is directly proportional to the officer force structure (i.e., the number of personnel tabulated by pay grade and length of service), the force structure's configuration must be forecasted accurately to predict future basic pay accurately.

Background

The officer force structure is characterized by 14 pay grades and 32 length-of-service (LOS) categories. A representative force structure matrix is depicted in Table 1. The pay grades are composed of the regular officer force (O-1 through O-10) and the four warrant officer grades (W-1 through W-4). The LOS category refers to the number of years in service, with cell 0 including all personnel with less than 1 year of service and cell 31 including all personnel with 31 or more years.

Once a force structure matrix is projected, a simple procedure is followed to obtain the cost associated with that configuration. First, because the force structure fluctuates over time, the projected population for a certain period of time is obtained by finding the average number of personnel within a pay category at the beginning and at the end of the period. Next, the cost is calculated by multiplying the average strength within each pay category by a prespecified statutory rate and summing over all pay categories. The 125 pay categories and a set of representative rates are depicted in Table 2. Not included in Table 2 are the pay rates for O-1's, O-2's, and O-3's with 4 or more years enlisted service. Naval Personnel Pay Predictor, Officer (NAPPO) does not attempt to forecast this particular group of personnel because future pay grade totals, which are input to NAPPO, do not include numbers for these personnel. The primary task, then, in forecasting the officer basic pay entails the forecasting of the force structure.

The NAPPO model employs several sources of data. First, four separate quarterly force structure files dating back to 1963 are maintained. The four files contain United States Navy (USN); United States Navy, Reserve (USNR); United States Navy, Temporary (USNT); and total force (ALNAV) data, the latter being the sum of the other three files for corresponding grades, years of service, and point in time. Thus, snapshots of those forces over time provide the basis for applying time series analysis. Also used by NAPPO for validation and forecasting purposes are (1) the pay tables (e.g., Table 2) that have been in effect since 1963, (2) managerial inputs in the form of pay grade totals for each quarter that is to be predicted, and (3) actual (historical) pay grade totals. If predictions are to be reliable, all of the data base must be kept as up-to-date as possible.

Objective

The objective of this report is to provide a detailed description of the analysis and procedures used to formulate the NAPPO model.

Table 1

A Typical Force Structure Matrix Showing Population
by Pay Grade and Longevity

| LOS | 0-1 | 0-2 | 0-3 | 0-4 | 0-5 | 0-6 | 0-7 | 0-8 | 0-9 | 0-10 | W-1 | W-2 | W-3 | W-4 | TOTAL |
|-------|------|------|-------|-------|------|------|-----|-----|-----|------|-----|------|-----|-----|-------|
| 0 | 2720 | 143 | 12 | 3 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2879 |
| 1 | 3047 | 265 | 46 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3359 |
| 2 | 706 | 2194 | 110 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3010 |
| 3 | 276 | 3068 | 130 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3474 |
| 4 | 131 | 1093 | 1868 | 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3114 |
| 5 | 107 | 416 | 2512 | 37 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 2 | 0 | 3076 |
| 6 | 172 | 214 | 2549 | 71 | 1 | 0 | 0 | 0 | 0 | 0 | 3 | 1 | 0 | 0 | 3011 |
| 7 | 246 | 273 | 2289 | 95 | 6 | 3 | 0 | 0 | 0 | 0 | 4 | 6 | 0 | 0 | 2922 |
| 8 | 192 | 298 | 1949 | 292 | 6 | 1 | 0 | 0 | 0 | 0 | 14 | 13 | 0 | 0 | 2757 |
| 9 | 161 | 422 | 1539 | 891 | 2 | 1 | 0 | 0 | 0 | 0 | 8 | 16 | 0 | 0 | 3040 |
| 10 | 136 | 293 | 718 | 1345 | 5 | 1 | 0 | 0 | 0 | 0 | 7 | 13 | 0 | 0 | 2518 |
| 11 | 110 | 143 | 543 | 1298 | 10 | 2 | 0 | 0 | 0 | 0 | 7 | 33 | 0 | 0 | 2146 |
| 12 | 120 | 83 | 360 | 1241 | 52 | 0 | 0 | 0 | 0 | 0 | 4 | 40 | 0 | 0 | 1900 |
| 13 | 97 | 69 | 282 | 1140 | 112 | 1 | 0 | 0 | 0 | 0 | 3 | 48 | 2 | 0 | 1754 |
| 14 | 70 | 55 | 290 | 1065 | 172 | 1 | 0 | 0 | 0 | 0 | 5 | 82 | 12 | 0 | 1752 |
| 15 | 66 | 66 | 270 | 815 | 443 | 4 | 0 | 0 | 0 | 0 | 2 | 162 | 26 | 0 | 1861 |
| 16 | 49 | 89 | 296 | 605 | 775 | 4 | 0 | 0 | 0 | 0 | 5 | 171 | 55 | 0 | 2049 |
| 17 | 32 | 117 | 284 | 504 | 767 | 5 | 0 | 0 | 0 | 0 | 2 | 217 | 86 | 5 | 2019 |
| 18 | 17 | 120 | 262 | 538 | 785 | 9 | 0 | 0 | 0 | 0 | 4 | 208 | 118 | 9 | 2070 |
| 19 | 6 | 109 | 264 | 569 | 866 | 41 | 0 | 0 | 0 | 0 | 7 | 192 | 134 | 40 | 2228 |
| 20 | 2 | 50 | 187 | 321 | 734 | 98 | 0 | 1 | 0 | 0 | 0 | 107 | 95 | 37 | 1632 |
| 21 | 1 | 44 | 178 | 241 | 664 | 255 | 0 | 0 | 0 | 0 | 1 | 113 | 75 | 59 | 1631 |
| 22 | 2 | 27 | 169 | 227 | 508 | 434 | 0 | 0 | 0 | 0 | 0 | 71 | 55 | 75 | 1568 |
| 23 | 1 | 21 | 95 | 208 | 337 | 413 | 1 | 0 | 0 | 0 | 0 | 44 | 20 | 71 | 1211 |
| 24 | 0 | 4 | 37 | 124 | 320 | 475 | 5 | 1 | 0 | 0 | 0 | 13 | 13 | 35 | 1027 |
| 25 | 0 | 6 | 48 | 163 | 216 | 483 | 9 | 1 | 0 | 0 | 0 | 8 | 20 | 70 | 1024 |
| 26 | 0 | 1 | 33 | 107 | 160 | 392 | 20 | 1 | 1 | 0 | 0 | 3 | 11 | 58 | 787 |
| 27 | 0 | 1 | 22 | 60 | 103 | 231 | 15 | 3 | 0 | 0 | 0 | 1 | 4 | 35 | 475 |
| 28 | 0 | 0 | 12 | 51 | 67 | 144 | 15 | 3 | 4 | 0 | 0 | 0 | 3 | 16 | 315 |
| 29 | 0 | 0 | 13 | 100 | 149 | 177 | 18 | 3 | 2 | 0 | 0 | 0 | 4 | 48 | 514 |
| 30 | 0 | 0 | 3 | 27 | 89 | 100 | 15 | 4 | 1 | 0 | 0 | 0 | 2 | 14 | 255 |
| 31 | 0 | 0 | 1 | 22 | 127 | 357 | 45 | 65 | 26 | 8 | 0 | 1 | 0 | 13 | 665 |
| TOTAL | 8467 | 9684 | 17363 | 12183 | 7476 | 3633 | 143 | 82 | 34 | 8 | 78 | 1570 | 737 | 585 | 62043 |

Table 2

A Typical Monthly Pay Matrix for Officer Personnel

| Pay Grade | Cumulative Years of Service | | | | | | | | | | | | | |
|-----------|-----------------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------------------|---------|----------------------|---------|----------|
| | Under 2 | Over 2 | Over 3 | Over 4 | Over 6 | Over 8 | Over 10 | Over 12 | Over 14 | Over 16 | Over 18 | Over 20 | Over 22 | Over 26 |
| O-10 | 3298.20 | 3414.30 | - | - | - | 3545.10 | - | 3816.90 | - | 4089.90 ^a | - | 4363.50 ^a | - | 4635.60* |
| O-9 | 2923.20 | 3000.00 | 3063.60 | - | - | 3141.90 | - | 3272.10 | - | 3545.10 | - | 3816.90 | - | 4089.90* |
| O-8 | 2647.50 | 2727.00 | 2791.80 | - | - | 3000.00 | - | 3141.90 | - | 3272.10 | 3414.30 | 3545.10 | 3687.30 | - |
| O-7 | 2199.90 | 2349.60 | - | - | 2549.90 | - | 2597.40 | - | 2727.00 | 3000.00 | 3206.10 | - | - | - |
| O-6 | 1630.50 | 1791.90 | 1908.60 | - | - | - | - | - | 1973.40 | 2286.00 | 2403.00 | 2454.90 | 2597.40 | 2817.00 |
| O-5 | 1304.40 | 1531.80 | 1637.40 | - | - | - | 1687.20 | 1777.50 | 1896.30 | 2038.50 | 2155.80 | 2220.60 | 2298.30 | - |
| O-4 | 1099.50 | 1338.30 | 1428.00 | - | 1454.10 | 1518.90 | 1622.10 | 1713.60 | 1791.90 | 1869.90 | 1922.10 | - | - | - |
| O-3 | 1021.80 | 1142.10 | 1220.70 | 1350.90 | 1415.40 | 1466.70 | 1545.30 | 1622.10 | - | - | - | - | - | - |
| O-2 | 890.70 | 972.90 | 1168.80 | 1208.10 | 1233.00 | - | - | - | - | - | - | - | - | - |
| O-1 | 773.10 | 804.90 | 972.90 | - | - | - | - | - | - | - | - | - | - | - |
| W-4 | 1040.70 | 1116.60 | - | 1142.10 | 1194.30 | 1246.80 | 1299.00 | 1389.90 | 1454.10 | 1505.70 | 1545.30 | 1596.00 | 1649.40 | 1777.50 |
| W-3 | 946.20 | 1026.30 | - | 1039.20 | 1051.50 | 1128.30 | 1194.30 | 1233.00 | 1272.30 | 1310.40 | 1350.90 | 1403.10 | 1454.10 | 1505.70 |
| W-2 | 828.60 | 896.10 | - | 922.20 | 972.90 | 1026.30 | 1065.00 | 1104.00 | 1142.10 | 1182.00 | 1220.70 | 1259.40 | 1310.40 | - |
| W-1 | 690.30 | 791.70 | - | 857.40 | 896.10 | 934.80 | 972.90 | 1013.10 | 1051.50 | 1090.20 | 1128.30 | 1168.80 | - | - |

^aLimited under existing law

APPROACH

The basic approach taken by NAPPO is an aggregative one. Although the use of the historical quarterly inventories allows ready observation of personnel flows, making direct predictions for each length of service (LOS) cell would require additional knowledge of promotions, such as time-in-grade and flow points. This information, in turn, would not only lead to the expansion of the data base but also to a much more complex model. Instead, NAPPO relies solely upon historical data to project the future. It forecasts the LOS marginal distribution by considering its statistical properties and then distributes the personnel over the authorized pay grade totals to arrive at the final force structure matrix. The following sections describe in detail the methods employed to forecast future inventories.

Forecasting the LOS Distribution

Examination of the quarterly force structure tables suggests two alternative approaches for predicting LOS cells 1 through 31. (Cell 0 is a special case and is discussed beginning on page 9.) The first involves the use of the actual cell populations, denoted $P_j(t)$ (i.e., the population with longevity j at time t , with t being measured in quarters). As will be demonstrated shortly, a forecast for LOS cell j at time t , $F_j(t)$, is calculated using actual and/or previous forecasts for the quarters through time $t-1$. That is,

$$F_j(t) = \hat{P}_j(t-1). \quad (1)$$

The second approach consists of using a set of transformed variables that measure the transition of the force. They are defined as net loss rates (r) and are given by

$$r_j(t) = (P_j(t) - P_{j+1}(t+m))/P_j(t), \quad (2)$$

for $j = 0, 1, \dots, 30$. (The loss rate for cell 31 is also a special case and is discussed on page 11.) Thus, $(1-r_j(t))$, or $C_j(t)$, is the continuance or transition rate; namely, the proportion of personnel in LOS cell j at time t that "move" to cell $j+1$ at time $t+m$. Since LOS is measured in years, m is set equal to 4 in NAPPO, thus providing yearly transition rates. A forecast for LOS cell $j+1$ at time t is then computed as

$$F_{j+1}(t) = \hat{C}_j(t-4)P_j(t-4) \quad (3)$$

where $\hat{C}_j(t-4)$ is the predicted rate of flow from $P_j(t-4)$ to $P_{j+1}(t)$. Because $P_j(t-4)$ is known, this method requires only the forecasting of the continuance rates.

Given the four data sources available in the USN, USNR, USNT, and ALNAV files, there are four straightforward ways of forecasting the ALNAV population of LOS cell $j+1$ at time t using the two approaches outlined above. They can be written as

$$F_{j+1}^1(t) = P_{j1}(t-4)\hat{C}_{j1}(t-4) + P_{j2}(t-4)\hat{C}_{j2}(t-4) + P_{j3}(t-4)\hat{C}_{j3}(t-4),$$

$$F_{j+1}^2(t) = P_{j4}(t-4)\hat{C}_{j4}(t-4),$$

$$F_{j+1}^3(t) = \hat{P}_{j+1,1}(t-1) + \hat{P}_{j+1,2}(t-1) + \hat{P}_{j+1,3}(t-1) \quad (4)$$

or

$$F_{j+1}^4(t) = \hat{P}_{j+1,4}(t-1)$$

where $P_{jk}(t)$ designates the actual number of personnel in cell j at time t for data set $k = 1, 2, 3, 4$ indicating USN, USNR, USNT, and ALNAV, respectively. Note that the first two forecasts use predicted continuance rates, whereas the latter two use actual population projections. Also, the first and third forecasts combine USN, USNR, and USNT predictions to give an ALNAV forecast.

As was done in the NAPPE model, time series analysis techniques (cf., Box & Jenkins, 1970; Brown, 1963) were applied to the four data sets to forecast the continuance rates. For each of the 124 time series (31 different loss rates for each of the four data files), each consisting of 58 data points, the rates were plotted, along with their autocorrelation and partial autocorrelation functions. Also, the first differences of the loss rates and their autocorrelation and partial autocorrelation functions were computed and plotted. Seasonality factors were identified by calculating the autocorrelation functions and their standard errors. Finally, a chi-square test was made on the first 24 autocorrelations to determine whether they could be distinguished from white noise. This procedure was used in an attempt to identify a general class of time series models that would be applicable to most or all of those of the individual time series while, at the same time, being conservative in the consumption of computer time.

Table 3 presents the series' seasonality factors and suggests one or more appropriate time series models based on the autocorrelation and partial autocorrelation functions. The terminology employed is that of Box and Jenkins (1970, Chapters 3 and 4), where a (p,d,q) autoregressive integrated moving average (ARIMA) model is differenced d times and contains p autoregressive and q moving average parameters. Thus, an entry in the table such as $"(1,1,0) \times 4"$ implies that a $(1,1,0)$ model be fitted to the series created by taking $Z_t - Z_{t-4}$ for $t \geq 5$, where Z_t is the t^{th} observation in the series. This step eliminates a seasonality factor of period 4.

The undifferenced data in a large majority of each set of series indicated non-stationary behavior. Many of these appear to be well suited to a single exponential smoothing model because, in general, they do not vary about a fixed mean, yet they exhibit homogeneous behavior of a kind. In this regard, Box and Jenkins (1970) comment: "... although the general level about which fluctuations are occurring may be different at different times, the broad behavior of the series, when differences in level are allowed for, may be similar" (p. 11). The exponential smoothing model is most easily recognized by its recursion formula,

$$\bar{Z}_t(\alpha) = \alpha Z_{t-1} + (1-\alpha)\bar{Z}_{t-1}(\alpha), \text{ for } 0 \leq \alpha \leq 1, \quad (5)$$

Table 3

Seasonality Factors and Appropriate Models for
Loss Rate Time Series

| LOS | USN Seasonality Factor | USN Suggested Appropriate Models | USNR Seasonality Factor | USNR Suggested Appropriate Models | USNT Seasonality Factor | USNT Suggested Appropriate Models | ALNAV Seasonality Factor | ALNAV Suggested Appropriate Models |
|-----|------------------------------|---|-------------------------------|--|-------------------------------|--|--------------------------------|---|
| 0 | NONE | (0,0,1) | NONE | (0,0,1) | NONE | (1,0,0) | 2 | (1,0,0);(0,1,0) x 2 |
| 1 | NONE | (0,0,1) | NONE | (0,0,1) | NONE | (0,1,1);(0,0,0) | NONE | (0,0,1) |
| 2 | NONE | (1,1,0);(0,2,0) | NONE | (0,1,0);(1,0,0) | NONE | (0,1,1);(0,0,0) | NONE | (0,1,0);(1,0,0) |
| 3 | NONE | (2,0,0);(0,1,0) | 4 | (1,0,0);(0,1,0) x 4 | NONE | (0,1,1);(0,0,0) | NONE | (1,0,0);(0,1,0) |
| 4 | NONE | (1,0,0);(0,1,0) | 4 | (0,1,0) x 4 | NONE | (0,1,1);(0,0,0) | NONE | (1,0,0);(0,1,1) |
| 5 | NONE | (1,0,0);(0,1,0) | 2 | (1,0,0);(0,1,0) x 2 | NONE | (0,1,1);(0,0,0) | 2 | (2,0,0);(0,1,0) x 2 |
| 6 | NONE | (1,0,0);(0,1,0) | NONE | (1,0,0);(0,1,0) | NONE | (0,1,1);(0,0,0) | NONE | (0,1,0);(1,0,0) |
| 7 | NONE | (2,0,0);(0,1,0) | NONE | (1,0,0);(0,1,0) | NONE | (0,0,1) | NONE | (0,1,0) |
| 8 | NONE | (1,0,0);(0,1,0) | 4 | (1,0,0);(0,1,0) x 4 | NONE | (0,0,1) | NONE | (0,0,1) |
| 9 | NONE | (1,0,0);(0,1,0) | NONE | (1,0,0);(0,1,1) | NONE | (0,0,1) | 4 | (1,0,0);(0,1,0) x 4 |
| 10 | NONE | (1,0,0);(0,1,0) | NONE | (1,0,0);(0,1,0) | NONE | (0,1,0);(1,0,0) | NONE | (0,1,1) |
| 11 | NONE | (1,0,0);(0,1,0) | NONE | (1,0,0);(0,1,0) | 4 | (1,0,0);(0,1,0) x 4 | 4 | (1,0,0);(0,1,0) x 4 |
| 12 | NONE | (1,0,0);(0,1,0) | 4 | (1,0,0);(0,1,0) x 4 | NONE | (0,1,0);(1,0,0) | 4 | (0,1,0) x 4 |
| 13 | NONE | (1,0,0);(0,1,0) | NONE | (1,0,0);(0,1,0) | NONE | (1,0,0);(0,1,0) | NONE | (0,1,0) |
| 14 | NONE | (1,0,0);(0,1,1) | NONE | (1,0,0);(0,1,0) | NONE | (1,0,0);(0,1,0) | NONE | (0,1,1) |
| 15 | 4 | (2,0,0);(0,1,0) x 4 | NONE | (1,0,0);(0,1,0) | NONE | (1,0,0);(0,1,0) x 4 | NONE | (1,0,0);(0,1,0) |
| 16 | 4 | (1,0,0);(0,1,0) x 4 | NONE | (1,0,0);(0,1,0) | 4 | (1,0,0);(0,1,0) x 4 | NONE | (0,1,1) |
| 17 | 4 | (1,0,0);(0,1,0) x 4 | NONE | (1,0,0);(0,1,0) | 4 | (1,0,0);(0,1,0) x 4 | NONE | (0,1,1) |
| 18 | NONE | (1,0,0);(0,1,0) | NONE | (1,0,0);(0,1,0) | 4 | (1,0,0);(0,1,0) x 4 | NONE | (0,1,1) |
| 19 | NONE | (1,0,0);(0,1,0) | NONE | (1,0,0);(0,1,0) | 4 | (1,0,0);(0,1,0) x 4 | 4 | (0,1,0) x 4 |
| 20 | NONE | (1,0,0);(0,1,0) | NONE | (1,0,0);(0,1,0) | 4 | (1,0,0);(0,1,0) x 4 | NONE | (0,1,0) |
| 21 | 4 | (1,0,0);(0,1,0) x 4 | NONE | (0,0,1);(0,1,0) | 4 | (1,0,0);(0,1,0) x 4 | NONE | (0,1,0) |
| 22 | NONE | (1,0,0);(0,1,0) | NONE | (1,0,0);(0,1,1) | 4 | (1,0,0);(0,1,0) x 4 | NONE | (0,1,0) |
| 23 | NONE | (1,0,0);(0,1,1) | 4 | (1,0,0);(0,1,0) x 4 | 4 | (1,0,0);(0,1,0) x 4 | NONE | (0,1,1) |
| 24 | NONE | (1,1,0);(2,0,0) | NONE | (0,0,1);(0,1,0) | NONE | (0,1,0) | NONE | (1,1,0) |
| 25 | 4 | (2,0,0);(0,1,0) x 4 | NONE | (1,0,0);(0,1,0) | NONE | (0,1,0) | NONE | (0,1,0) |
| 26 | 4 | (2,0,0);(0,1,0) x 4 | NONE | (0,0,1);(0,1,1) | NONE | (0,1,1) | NONE | (0,1,0) |
| 27 | NONE | (0,0,0);(0,1,0) | NONE | (1,0,0);(0,1,1) | NONE | (0,1,0) | NONE | (0,1,0) |
| 28 | 2 | (0,0,1);(0,1,0) x 2 | NONE | (1,0,0);(0,0,1) | 2 | (0,1,0) x 2 | 2 | (0,1,0) x 2 |
| 29 | NONE | (1,0,0);(0,1,0) | NONE | (0,0,1);(0,1,0) | NONE | (0,1,0) | NONE | (0,1,0) |

which shows that each new level of the process (the forecast), $\bar{Z}_t(\alpha)$, is arrived at by interpolating between the new observation, Z_{t-1} , and the previous level, $\bar{Z}_{t-1}(\alpha)$. With α at its upper limit of 1, the level of the process is simply the latest observation, with all previous history ignored. With α small (closer to 0), the previous level is given more weight than the latest observation. The smaller α is, the longer the time period required by the model to recognize a change in "levels" of the data.

Seventy-seven of the time series seem to be modelled well by single exponential smoothing. First, 56 of the series were reduced to white noise after first differencing implying a (0,1,0) model (i.e., $Z_t = Z_{t-1} + a_t$ where a_t is the noise component and Z_t is the t^{th} observation of the series). But this is simply equation (5) with α equal to 1. As shown in Table 3, these series were USN cells 3-6, 7-13, 18-20, 22, 27, 29; USNR cells 2, 6, 7, 10, 11, 13-21, 24, 25, 29, 30; USNT cells 10, 12-15, 24, 25, 27, 29, 30; and ALNAV cells 2, 3, 6, 7, 13, 15, 20-22, 25-27, 29. It was indicated that another 20 series (USN cells 14, 23; USNR cells 9, 22, 26, 27; USNT cells 1-6, 26; and ALNAV cells 4, 10, 14, 16-18, 23) could be fitted by a (0,1,1) ARIMA process (i.e., $Z_t = Z_{t-1} + a_t - \theta a_{t-1}$, where $\theta = 1 - \alpha$). Since this is the inverted form of equation (5) (see Box & Jenkins, pp. 105-106), however, these series can also be forecasted using the single exponential smoothing model. Note that double and triple exponential smoothing are not applicable here because no evidence exists of any linear or quadratic trends in the data.

Another 32 series, which required first differencing to achieve stationarity, contained evidence of a seasonality factor. A significant autocorrelation at lag 2 was noticed in 6 of these series (USN cell 28; USNR cell 5; USNT cell 28; and ALNAV cells 0, 5, 28). Here, a significant autocorrelation at lag n means that pairs of observations separated by n time periods are significantly correlated. Twenty-six series possessed an indication of a cycle of length 4 (USN cells 15-17, 21, 25, 26, 30; USNR cells 3, 4, 8, 12, 23; USNT cells 11, 16-23; and ALNAV cells 9, 11, 12, 19, 30).

Finally, 66 of the series could also be modelled well by an autoregressive model of order 1, the (1,0,0) model. In notational form, the model is $Z_t = \phi Z_{t-1} + a_t$, where $-1 < \phi < 1$. Table 3 indicates the 66 series to be USN cells 4-6, 8-23, 27, 29; USNR cells 2, 3, 5-18, 20, 22, 23, 25, 27, 28; USNT cells 0, 10-23; and ALNAV cells 0, 2-4, 6, 9, 11, 15. Some of these cells are such that two or more time series models were suggested by the autocorrelation functions. This was the case with many of the (1,0,0) cells.

Because the single exponential smoothing model, the seasonal model of period 4, and the first order autoregressive model adequately explain 26 of the USN series, 29 of the USNR series, 27 of the USNT series, and 26 of the ALNAV series, no further models for the loss rates were considered. The remaining 18 series not covered by these models are explained by an assortment of various models, including the (0,0,1), (2,0,0), and (1,1,0) ARIMA processes. Since each of these models requires the estimations of at least one parameter, increased forecasting accuracy was sacrificed in the interest of using less computer time. It should be noted that the α parameter in the exponential smoothing model is estimated by choosing that α from the set $\{0, .05, .10, \dots, .95, 1.0\}$ that minimizes historical error. The same is true for the parameter ϕ in the single order autoregressive model although, technically speaking, ϕ could vary from -1 to 1. Negative values for ϕ were excluded since no negative autocorrelations of lag 1 were observed in the time series. In effect, then, 21 exponential smoothing models (for 21 α 's), 21 autoregressive models (for 21 ϕ 's), and 1 seasonal model (requiring no parameter estimation) are fitted to all 124 time series and the model that best explains the past is chosen to forecast future transition rates.

Similar procedures were used to produce a set of models that would fit most of the actual population data. Table 4 summarizes the analysis for the 124 time series covering LOS cells 1-31. (Cell 0 is a special case and is discussed below). In contrast to the enlisted LOS cell populations, very little seasonality is evident in the officer populations. Ninety-two of the 124 time series are best modelled by the single exponential, first order autoregressive, or seasonal model of period 4, all of which are described above. The exponential model alone appears to be adequate for 79 of those.

USN, USNR, USNT, and ALNAV forecasts for a particular quarter are provided for each LOS cell using continuance rates and actual population data. These individual forecasts are produced by the model that minimizes historical error. These eight forecasts for each cell (two forecasts for a particular LOS cell in each of four data sets) are used to produce the four ALNAV forecasts seen in equation (4), where the USN, USNR, and USNT forecasts are combined.

Examination of the forecasts F^1 through F^4 in equation (4) showed that the forecasts based on continuance rates (F^1, F^2) yielded less historical error than those based on actual population data (F^3, F^4). Consequently, in some cases, the latter forecasts could be used to "damp" the continuance rate forecasts. Such instances would be characterized by conditions in which the continuance rate varies inversely with the source population. Thus, a fifth prediction was made by taking a weighted sum of the first four:

$$F_{j+1}^5(t) = \beta(F_{j+1}^1(t) + F_{j+1}^2(t))/2 + (1-\beta)(F_{j+1}^3(t) + F_{j+1}^4(t))/2, \quad (6)$$

$0 \leq \beta \leq 1$. Here β is some weight that produces a minimum sum of squared error. Finally, the forecast for a particular cell is the one chosen from $\{F^1, F^2, F^3, F^4, F^5\}$ that has been the best estimator of the past, with "best" implying the forecast that minimizes historical error. As will be seen, this "final" forecast will most likely be modified.

Special Considerations for Cells 31 and 0

As noted earlier, LOS cell 31 contains all personnel with more than 31 years of service. Because the continuance rate as previously defined only explains the movement of personnel from cell 30 to cell 31, a special rate was defined that would encompass the movement of personnel from cell 30 as well as the continuance of those personnel already in cell 31. It is written

$$r_{31}(t) = ((P_{30}(t) + P_{31}(t)) - P_{31}(t+4))/(P_{30}(t) + P_{31}(t)). \quad (7)$$

(Compare with (2).) The time series composed of these rates was analyzed in the same fashion as those for $r_0(t)$ through $r_{30}(t)$.

Cell 0 is also a special case; that is, no continuance rate can be defined for it because there is obviously no source population with military service less than 0 years. Also, due to the fluctuating input of newly commissioned officers (determined largely by personnel policies), the high degree of instability of populations in cell 0 make it undesirable to smooth the historical populations in that cell. Yet, one ready-made forecast is available from the realization that the sum of the forecasts for cells 0 through 31 must equal the total force size. Thus, we have the "residual forecast":

Table 4

Seasonality Factors and Appropriate Models for
Actual Population Time Series

| LOS | Seasonality Factor | USN Appropriate Model | Seasonality Factor | USNR Appropriate Model | Seasonality Factor | USNT Appropriate Model | Seasonality Factor | ALNAV Appropriate Model |
|-----|--------------------|--------------------------|--------------------|---------------------------|--------------------|---------------------------|--------------------|----------------------------|
| 1 | NONE | (0,1,0) | NONE | (0,1,1) | NONE | (1,0,0); (0,1,0) x 2 | NONE | (0,1,0) |
| 2 | 2 | (0,1,0) x 2 | NONE | (2,1,0) | NONE | (0,0,1) | NONE | (2,1,0) |
| 3 | 3 | (0,1,0) x 3 | NONE | (0,1,1) | NONE | (0,0,1); (0,1,0) | NONE | (0,1,1) |
| 4 | 4 | (0,1,0) x 4 | 4 | (1,0,0); (0,1,0) x 4 | NONE | (0,0,0) | NONE | (1,0,0) |
| 5 | NONE | (0,1,0) | NONE | (0,1,1) | NONE | (0,0,2) | NONE | (1,0,0); (2,0,0) |
| 6 | NONE | (0,1,0) | NONE | (0,1,1) | NONE | (0,0,1) | NONE | (0,1,1) |
| 7 | NONE | (0,1,0) | NONE | (0,1,1) | NONE | (0,1,0) | NONE | (1,0,1); (0,1,0) |
| 8 | NONE | (0,1,0) | NONE | (0,1,0) | NONE | (0,1,0) | NONE | (1,0,0); (0,1,0) |
| 9 | 4 | | NONE | (0,1,0) | NONE | (0,0,1) | NONE | (0,0,2) |
| 10 | NONE | (0,0,2); (1,0,0) | NONE | (0,1,0) | NONE | (0,1,0) | NONE | (2,0,1); (1,0,1) |
| 11 | NONE | (2,0,1); (1,0,1) | NONE | (0,1,0) | NONE | (0,1,0) | 4 | (1,0,0); (0,1,0) x 4 |
| 12 | NONE | (1,0,0) | NONE | (0,1,0) | NONE | (0,1,0) | NONE | (1,0,0); (0,1,1) |
| 13 | NONE | (1,0,0); (0,1,1) | NONE | (0,1,1) | NONE | (0,1,1) | NONE | (1,0,0); (0,1,1) |
| 14 | NONE | (0,1,1) | NONE | (0,1,1) | NONE | (2,0,0) | NONE | (0,1,1) |
| 15 | NONE | (0,1,1) | NONE | (0,1,0) | NONE | (2,0,0) | NONE | (1,0,0); (0,1,1) |
| 16 | NONE | (0,1,1) | NONE | (0,1,1) | NONE | (2,0,0) | NONE | (2,0,0) |
| 17 | NONE | (0,1,1) | NONE | (0,1,1) | NONE | (2,0,0) | NONE | (2,0,0) |
| 18 | NONE | (0,1,1) | NONE | (0,1,1) | NONE | (2,0,0) | NONE | (2,0,0) |
| 19 | NONE | (0,1,1) | NONE | (1,1,0); (0,1,1) | NONE | (2,0,0) | NONE | (2,0,0) |
| 20 | NONE | (0,1,2); (1,0,0) | NONE | (0,1,0) | NONE | (2,0,0) | NONE | (0,1,2); (1,0,0) |
| 21 | NONE | (1,0,0); (0,1,1) | NONE | (1,0,0); (0,1,1) | NONE | (2,0,0); (2,0,1) | NONE | (0,1,1); (1,0,0) |
| 22 | NONE | (0,1,0) | NONE | (1,0,0); (0,1,1) | NONE | (2,1,0); (2,0,0) | NONE | (0,1,1); (1,0,0) |
| 23 | NONE | (0,1,2) | NONE | (1,0,0); (0,1,1) | NONE | (0,1,1) | NONE | (0,1,1) |
| 24 | NONE | (0,1,2) | NONE | (1,0,0); (0,1,1) | NONE | (0,1,2); (1,1,0) | NONE | (0,1,1) |
| 25 | NONE | (0,1,1) | NONE | (2,0,0); (0,1,1) | NONE | (1,1,0) | NONE | (0,1,1) |
| 26 | NONE | (0,1,1); (0,1,2) | NONE | (2,0,0); (0,1,1) | NONE | (1,1,0) | NONE | (0,1,1) |
| 27 | NONE | (0,1,1) | NONE | (0,1,1) | NONE | (1,1,0) | NONE | (0,1,1) |
| 28 | NONE | (0,1,1) | NONE | (0,1,1) | NONE | (1,1,0) | NONE | (0,1,1) |
| 29 | NONE | (0,1,1) | NONE | (0,1,1) | NONE | (1,1,0) | NONE | (0,1,1) |
| 30 | NONE | (0,1,1) | NONE | (0,1,1) | NONE | (0,1,0) | NONE | (0,1,1) |
| 31 | 2 | (0,1,0) x 2 | NONE | (0,1,0) | NONE | (1,1,0) | 2 | (0,1,0) x 2 |

$$\hat{P}_0(t) = F(t) - \sum_{j=1}^{31} \hat{P}_j(t), \quad (8)$$

where \hat{P}_0 is the ALNAV forecast for cell 0, F is the total force, and \hat{P}_j is the best forecast selected from $\{F^1, F^2, F^3, F^4, F^5\}$ for LOS cell j . This forecast for cell 0 may be abnormally small, large, or even negative, however, due to large changes in the total force size and the relative stability of forecasts \hat{P}_1 through \hat{P}_{31} . Therefore, the following analysis was performed to obtain more realistic forecasts for all the \hat{P}_j 's.

First, proportions were computed that relate the size of cell zero to the total force size over the historical data base. Symbolically, these ratios, denoted R_i , are determined by

$$R_i = P_0(i)/F(i), \text{ for } i = 1, \dots, 62 \quad (9)$$

where i represents 62 quarters of history and $P_0(i)$ and $F(i)$ are the cell 0 and total force populations for quarter i . Next, the R_i 's were examined for trends and patterns using time series analysis. As it turned out, a second-order moving average process applied to first differences fits the data extremely well. This model is written $Z_t = Z_{t-1} - \phi a_{t-2}$, where a_{t-2} , the error for period $t-2$, is the actual value minus the forecasted value for that period and ϕ lies between 1 and -1. Thus, the forecast for a cell zero to total force ratio for time period t , denoted R_t , is determined from

$$\hat{R}_t = R_{t-1} - .5666a_{t-2} \quad (10)$$

or

$$\hat{R}_t = R_{t-1} - .5666(R_{t-2} - \hat{R}_{t-2}).$$

Here $\phi = .5666$ and \hat{R}_{t-2} is the forecast for time period $t-2$. Applying the forecasted ratio to the user-supplied total officer force yields another forecast for the cell 0 population. In addition, 95 percent confidence limits for \hat{R}_t can be derived by substituting the 95 percent confidence limits (.34259, .79061) for the value of ϕ in equation (10).

The final formula for computing the cell 0 population for any future quarter t uses the residual forecast \hat{P}_0 for that quarter as well as equation (10):

$$\hat{P}_0^*(t) = \max\{(R_t - .79061a_{t-2}) * F(t), \min\left(\frac{(P_0(t) + (R_t - .5666a_{t-2}) * F(t))}{2}, (R_t - .34259a_{t-2}) * F(t)\right)\} \quad (11)$$

where the first term in the brackets is the lower bound on the size of cell 0; the second, the average of the residual forecast and the moving average forecast; and the third, the upper bound on the size of cell 0. The lower and upper bounds in equation (11) assume $a_{t-2} > 0$. If $a_{t-2} < 0$, the coefficients .34259 and .79061 would be interchanged.

It is quite likely, however, that the adjusted cell 0 forecast, $\hat{P}_0^*(t)$, is different from $\hat{P}_0(t)$, the residual forecast. Thus, the discrepancy, $\hat{P}_0(t) - \hat{P}_0^*(t)$, must be distributed over each of the remaining cells. The procedure used ensures that the cells that have been predicted with a high degree of accuracy are altered only slightly. It distributes the discrepancy according to the amount of the prediction error in a cell relative to the total prediction error over all other cells:

$$\hat{P}_j^*(t) = \hat{P}_j(t) + (\hat{P}_0(t) - \hat{P}_0^*(t)) * (E_j(t) \hat{P}_j(t) / \sum_{j=1}^{31} E_j(t) \hat{P}_j(t)), \quad (12)$$

$j = 1, 2, \dots, 31$, where the error is calculated as

$$E_j(t) = \sum_{i=5}^{t-1} |P_{j4}(i) - \hat{P}_j(i)| / P_{j4}(i) \quad (13)$$

for cell j . Recall that $P_{j4}(i)$ denotes the actual ALNAV population for cell j in quarter i . Also note that four quarters are required to initiate the forecasting process (for computation of continuance rates); hence, the lower range in the summation.

Having executed the above procedure, NAPPO now has the LOS marginal distribution $(\hat{P}_0^*, \hat{P}_1^*, \hat{P}_2^*, \dots, \hat{P}_{31}^*)$ of the force structure matrix. The vector is complete and sums to the total authorized strength. The pay grade vector is either supplied by the user or interpolated by NAPPO (discussed on page 14). Thus, the problem now is to calculate the interior entries in the force structure matrix. Once this has been accomplished, multiquarter forecasts are obtained by concatenating the predicted quarter with the historical data base and employing the optimal time series models that were chosen for the first prediction.

Constructing the Force Structure Matrix

Given the two marginal distributions of the force structure matrix, NAPPO follows Mosteller's (1968) procedure for deriving the interior. This iterative procedure involves the renormalization of the rows and columns of a standard matrix to obtain the given marginals, while maintaining the relative "associations" among the entries of the standard matrix. In NAPPO, the standard matrix is computed as the average of the previous 12 historical ALNAV inventories. Although the Mosteller procedure is documented in the NAPPE report (Chipman, 1977), it is presented here again for convenience.

To begin, suppose that the LOS and pay grade marginals are denoted by L_i and G_j , respectively, where $i = 0, 1, \dots, 31$ and $j = 1, 2, \dots, 14$. With these and the standard matrix being given, Mosteller's procedure involves the following steps:

1. Let $n = 1$.
2. Let $S_{ij}^{n+1} = (L_i / \sum_j S_{ij}^n) S_{ij}^n$ for all i .
3. Let $S_{ij}^{n+2} = (G_j / \sum_i S_{ij}^{n+1}) S_{ij}^{n+1}$ for all j .
4. If $\sum_j S_{ij}^{n+2} \neq L_i$ for all i go to (2), otherwise stop.

This process allows estimates of a force structure matrix to be made for specific points in time. To predict basic pay costs for a fiscal year, some estimate of the configuration of that matrix throughout the year is necessary.

Estimating Average Strength

Given an actual inventory at some quarterly interval and a forecast inventory representing a period four quarters later, an estimate of average strength can be computed; that is,

$$\bar{S}_{ij} = (S_{ij} + \hat{S}_{ij})/2, \quad (14)$$

where S_{ij} and \hat{S}_{ij} represent the actual and forecasted strength matrix entries respectively. Clearly, the average force \bar{S}_{ij} derived in this manner assumes that changes in a force configuration over four quarters occur in a constant relation to time. Thus, if the difference between $\sum_{ij} S_{ij}$ and $\sum_{ij} \hat{S}_{ij}$ were 100, an average matrix would reflect a monthly change of 8.33 or a quarterly change of 25, etc.

Straightline methods are inadequate, however, because changes do not occur uniformly throughout the year. Unfortunately, the only practical alternatives in terms of available data are average strength computations based on quarterly increments. Thus, given S_{ij} for $t = \text{December } 1975$, a forecast of \hat{S}_{ij} is made for $t = \text{December } 1976$. Similarly, four-quarter forecasts are made June to June, September to September, and March to March. So, by forming force structure matrices for each quarterly prediction and by combining those four forecasts with the most recent actual matrix, a relatively good estimate of a yearly average strength can be computed by a five-point average:

$$\bar{S}_{ij} = (S_{ijt_0} + \hat{S}_{ijt_1} + \hat{S}_{ijt_2} + \hat{S}_{ijt_3} + \hat{S}_{ijt_4})/5. \quad (15)$$

Up to this point, estimates of future pay grade distributions were assumed to be an exogenous input accessible to the users of the model. The estimation of the 14 cells of the pay grade marginal distribution, however, is no trivial task.

While the end-year pay grade structure is roughly established by the budget (authorized end strength), the quarterly pay grade distributions are largely a function of management actions taken throughout the fiscal year. If pay grade totals for September, December, March, and June are input by the user, the model will form complete force structure matrices based on those given pay grade marginals and the forecasted longevity marginals. This input will be based on official plans that reflect management intentions. Specifically, the pay grade populations for the 0-3 through 0-10 and W-2 through W-4 mainly reflect the planned phasing of promotions during the subject year, while 0-1, 0-2, and W-1 primarily reflect the planned phasing of input.

If, instead, the pay grade totals for the three quarters interior to the fiscal year (the beginning total is known and the ending total is always input as a matter of strength/budget policy) are to be forecasted by NAPPO, the following procedures, akin to those in NAPPE, are used: Let $\hat{F}_{t,i}$ and $\hat{F}_{t+4,i}$ be the actual or user input pay grade totals for the end (or beginning) of 2 consecutive fiscal years. Estimates of the quarterly pay grade totals are then given by

$$\begin{aligned}\hat{F}_{t+2,i} &= (\hat{F}_{t,i} + \hat{F}_{t+4,i})/2, \\ \hat{F}_{t+1,i} &= (\hat{F}_{t,i} + \hat{F}_{t+2,i})/2,\end{aligned}\tag{16}$$

and

$$\hat{F}_{t+3,i} = (\hat{F}_{t+2,i} + \hat{F}_{t+4,i})/2.$$

These simple linear interpolations could be altered by increasing or decreasing them from the straightline results by reference to an average of historical seasonal patterns of pay grade population movement. Since analysis revealed wide variability in the quarterly deviations from straightline averages from year to year, however, it was decided to allow the user to implement any seasonal pattern deemed appropriate by directly inputting the quarterly future pay grade totals. Otherwise, NAPPO will use those obtained in equation (16).

Costing the Force Structure

The procedures outlined previously produce a forecast of the force structure matrix for the end of each quarter (31 December, 31 March, 30 June, and 30 September) during the forecast year. In addition, the beginning matrix (30 September) is available in the form of known (i.e., historical) data.

The following costing procedures are observed. If the pay scale is constant for the year, the five matrices or inventories can be averaged and a pay scale such as that in Table 2 applied to the average force to produce a forecast of annual basic pay costs. If the pay scale changes during a fiscal year, some provision must be made for costing part of the average strength with one pay scale and for costing its complement with another. Since no data are available to forecast population changes within a quarter, it is assumed that intraquarter changes occur uniformly throughout the quarter. Also, if the pay scale is in effect for a full quarter, the average strength for that quarter is defined by the average of the begin and end quarter matrices, and the cost is derived by applying the constant pay rates to the quarterly average. Finally, if the pay scale changes during the

quarter, then (1) the force structure at the point of change is approximated by linear interpolation between the begin and end quarter matrices, (2) the populations for the start and end of each pay scale are averaged to obtain the average force to which the relevant pay scale is applied, and (3) a corresponding daily pay scale is applied to the average force. The results then are multiplied by the number of days for which the pay scale is applicable. The results for all quarters--whether single or dual pay scales--are summed over the fiscal year to produce an estimate of annual basic pay.

This method is followed for any fiscal year to be predicted. It requires only the end strength pay grade totals for each year to be forecasted. The user may supply quarterly pay grade totals as desired.

RESULTS

Historical Validation of the NAPPO Model

One problem in obtaining a complete validation of NAPPO lies in the fact that the authorized end strengths are seldom achieved exactly. Consequently, although some discrepancy may exist between the actual and the forecasted force structures, it would not significantly affect the validation results.

To demonstrate NAPPO's accuracy, the model was used to forecast historical force structures and their respective costs for the fiscal years 1974-1978. NAPPO was allowed to use historical data up to 1, 2, and 3 years preceding the year to be predicted. All pay grade totals used were actual quarterly totals as opposed to authorized end strength totals, which may or may not have been achieved. It should be noted that, although the forecast for any given cell in the force structure matrix could be erroneous, the total cost may be accurate due to offsetting errors in other cells.

Therefore, an additional validation criterion was established to test for systematic bias in the forecasting process. This involved comparing the forecasted and actual longevity distributions, with the criterion being average length of service. Table 5 summarizes these results, given the three different lead times. As shown, there appears to be no evidence of consistent under- or overestimation of mean LOS, although, in general, the forecasts are less accurate as lead time increases.

The historical validation for basic pay involved the comparison of the costs of the projected and the actual force structures. The absolute values of the percentage differences between the two also are included in Table 5, which demonstrates NAPPO's accuracy for forecasts made 1, 2, and 3 years in advance of the projected period. Overall, the average absolute 1-year lead time fiscal year percent error in cost prediction in Table 5 is .21 percent, with a standard deviation of .68 percent. Similar figures for the 2-year and 3-year lead time forecasts are .64 (1.55) and 1.06 (1.96) respectively. In forecasting mean LOS, the average fiscal year absolute error for the 1-, 2-, and 3-year lead times are .09, .25, and .45 years respectively. Once again, for both cost and mean LOS forecasts, accuracy generally diminishes with an increase in lead time.

NAPPO vs. Current Procedures

One means of measuring the usefulness of NAPPO is to compare the NAPPO forecasts with those in the annual budget submissions. Each year the budget submission gives 1- and 2-year forecasts for all of the Military Personnel, Navy budget activities, including officer pays and allowances.

Figure 1 depicts the percentage differences of the NAPPO and budget submission 2-year forecasts from the actual obligations incurred. In all cases NAPPO was more accurate, with an average absolute error of .64 percent, as compared to 4.00 percent for the budget submissions. The 1-year forecast errors for the same years were .18 percent for NAPPO and 1.40 percent for the submissions. When the data for FY71 and FY72 (the poor forecasting years for the submissions) are excluded, NAPPO is still, on the average, 3-1/2 times more accurate than the submission in 2-year forecasts of officer basic pay (.53% vs. 1.96%). In FY78 dollars, the average errors represent differences of \$5.8 million and \$21.6 million respectively.

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Table 5

Validation: Predicted Mean LOS and Percent Error in Cost Prediction
Based on Actual Quarterly Pay Grade Totals

| Lead Time Quarter | 1 Yr. Lead Time | | | 2 Yrs. Lead Time | | | 3 Yrs. Lead Time | | | Actual Mean LOS |
|----------------------|-----------------------|--------------------------|--|-----------------------|--------------------------|--|-----------------------|--------------------------|--|--------------------|
| | Predicted Mean LOS | % Error Cost Pred. | | Predicted Mean LOS | % Error Cost Pred. | | Predicted Mean LOS | % Error Cost Pred. | | |
| 9/69 | 10.82 | +27 | | | | | | | | 10.73 |
| 12/69 | 10.93 | +47 | | | | | | | | 11.07 |
| 3/70 | 10.94 | +43 | | | | | | | | 11.09 |
| 6/70 | 10.90 | +49 | | | | | | | | 10.93 |
| FY 70 | 10.90 | +42 | | | | | | | | 10.95 |
| 9/70 | 10.78 | -30 | | 10.86 | +51 | | | | | 10.98 |
| 12/70 | 10.79 | -81 | | 10.78 | +33 | | | | | 10.95 |
| 3/71 | 10.85 | -88 | | 10.80 | +03 | | | | | 11.04 |
| 6/71 | 10.78 | -92 | | 10.77 | -27 | | | | | 11.08 |
| FY 71 | 10.83 | -73 | | 10.82 | +14 | | | | | 11.00 |
| 9/71 | 11.13 | +04 | | 10.57 | -140 | | 10.86 | -35 | | 11.15 |
| 12/71 | 10.98 | -03 | | 10.50 | -188 | | 10.74 | -43 | | 11.12 |
| 3/72 | 11.13 | -09 | | 10.59 | -187 | | 10.78 | -66 | | 11.24 |
| 6/72 | 11.07 | -18 | | 10.49 | -197 | | 10.66 | -97 | | 11.17 |
| FY 72 | 11.08 | -07 | | 10.59 | -179 | | 10.76 | -61 | | 11.15 |
| 9/72 | 11.33 | +09 | | 11.07 | -41 | | 10.44 | -28 | | 11.31 |
| 12/72 | 11.26 | +09 | | 10.85 | -69 | | 10.36 | -53 | | 11.36 |
| 3/73 | 11.31 | -03 | | 10.93 | -88 | | 10.35 | -66 | | 11.40 |
| 6/73 | 11.29 | -14 | | 10.86 | -104 | | 10.24 | -86 | | 11.30 |
| FY 73 | 11.27 | -00 | | 10.95 | -76 | | 10.38 | -59 | | 11.31 |

Table 5 (Continued)

| Lead Time Quarter | 1 Yr. Lead Time | | | 2 Yrs. Lead Time | | | 3 Yrs. Lead Time | | | Actual Mean LOS |
|----------------------|-----------------------|--------------------------|--------------------------|-----------------------|--------------------------|--------------------------|-----------------------|--------------------------|--------------------------|--------------------|
| | Predicted Mean LOS | % Error Cost Pred. | % Error Cost Pred. | Predicted Mean LOS | % Error Cost Pred. | % Error Cost Pred. | Predicted Mean LOS | % Error Cost Pred. | % Error Cost Pred. | |
| 9/73 | 11.57 | +14 | | 11.37 | -27 | | 11.14 | -1.06 | | 11.39 |
| 12/73 | 11.46 | +07 | | 11.27 | -52 | | 10.94 | -1.18 | | 11.42 |
| 3/74 | 11.54 | -13 | | 11.32 | -74 | | 11.06 | -1.42 | | 11.50 |
| 6/74 | 11.48 | -05 | | 11.36 | -66 | | 10.96 | -1.42 | | 11.36 |
| FY 74 | 11.47 | +01 | | 11.32 | -55 | | 10.99 | -1.27 | | 11.39 |
| 9/74 | 11.51 | -01 | | 11.61 | +16 | | 11.54 | -43 | | 11.36 |
| 12/74 | 11.47 | -12 | | 11.37 | +03 | | 11.41 | -43 | | 11.34 |
| 3/75 | 11.60 | -13 | | 11.35 | -28 | | 11.40 | -58 | | 11.40 |
| 6/75 | 11.36 | -36 | | 11.35 | -12 | | 11.48 | -53 | | 11.33 |
| FY 75 | 11.46 | -16 | | 11.43 | -06 | | 11.44 | -50 | | 11.36 |
| 9/75 | 11.42 | +14 | | 11.23 | -52 | | 11.68 | +29 | | 11.26 |
| 12/75 | 11.49 | +28 | | 11.15 | -58 | | 11.53 | +35 | | 11.37 |
| 3/76 | 11.60 | +19 | | 11.41 | -75 | | 11.51 | +00 | | 11.52 |
| 6/76 | 11.50 | -02 | | 11.07 | -111 | | 11.52 | -21 | | 11.52 |
| FY 76 | 11.47 | +15 | | 11.25 | -75 | | 11.52 | +11 | | 11.40 |
| 9/76 (TQ) | 11.43 | -09 | | 11.33 | -27 | | 11.21 | -1.29 | | 11.54 |
| 12/76 | 11.64 | +08 | | 11.14 | -70 | | 11.17 | -1.11 | | 11.57 |
| 3/77 | 11.66 | +13 | | 11.20 | -76 | | 11.27 | -90 | | 11.65 |
| 6/77 | 11.64 | -20 | | 11.03 | -1.20 | | 11.01 | -1.325 | | 11.85 |
| 9/77 | 11.72 | -19 | | 11.03 | -1.39 | | 11.06 | -1.51 | | 11.68 |
| FY 77 | 11.64 | -05 | | 11.12 | -1.01 | | 11.12 | -1.21 | | 11.66 |
| 12/77 | 11.74 | +05 | | 11.67 | +13 | | 11.17 | -1.01 | | 11.68 |
| 3/78 | 11.73 | +07 | | 11.66 | +11 | | 11.18 | -87 | | 11.73 |
| 6/78 | 11.55 | +29 | | 11.65 | -27 | | 11.06 | -1.30 | | 11.97 |
| 9/78 | 11.30 | +74 | | 11.66 | -23 | | 11.00 | -1.38 | | 11.69 |
| FY 78 | 12.00 | +29 | | 11.67 | -07 | | 11.09 | -1.14 | | 11.75 |

NOTE: FY78 OFFICER BASIC PAY:\$1,093,603,000

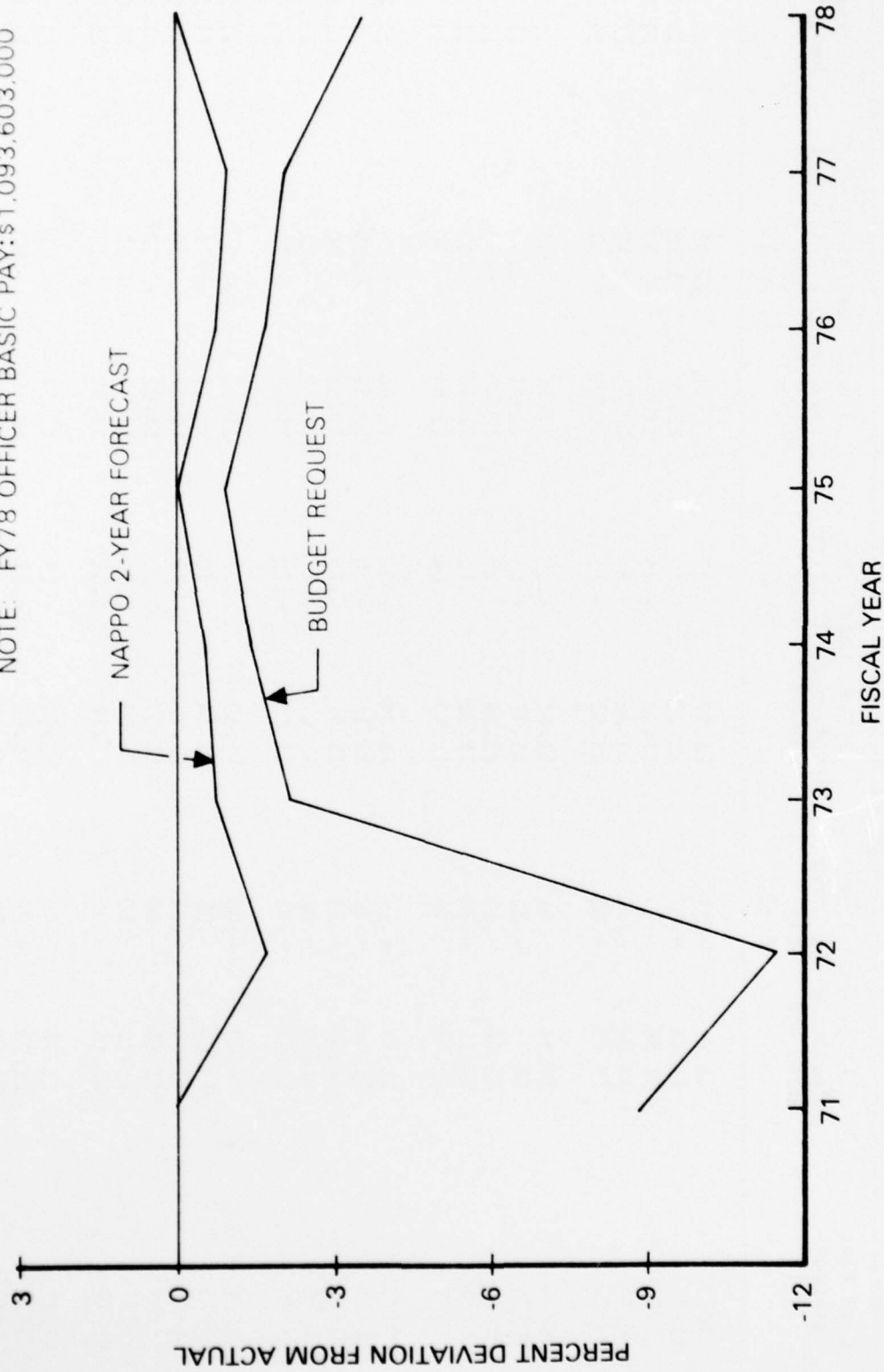


Figure 1. Error in estimating basic pay for Navy officer personnel.

The Implementation of NAPPO

NAPPO is currently operating in a demand mode on a UNIVAC 1110 computer located at the Naval Ocean Systems Center, San Diego. Currently, plans call for NAPPO to be included, along with NAPPE, in a comprehensive budget forecasting model. This model will provide forecasts for all budget activities of the Military Personnel, Navy account, including items such as enlisted and officer basic pay, enlisted and officer allowances, subsistence, and permanent change of station pay. A future report will document the all-MPN model, particularly the techniques used to forecast the budget items other than basic pay.

As a stand-alone model, NAPPO is useful not only for forecasting basic pay but also for monitoring the age of the force through the mean LOS forecasts. Thus, Navy management can be kept aware of upward or downward trends in the experience of the force. Also, for general use, NAPPO subroutines allow the user to forecast LOS transition rates, to update the historical data base, and to validate the model with respect to data ranging back to 1963. Still other routines permit the examination of historical transition rates or the force structure matrices themselves.

CONCLUSIONS

1. Sufficient data are available to allow time series analysis techniques to be applied toward the development of a model to forecast the officer force structure and, thus, basic pay obligations.

2. A particular set of time series models exists that adequately explains a great majority of the data. This set of models provides a solid statistical basis for making force structure forecasts.

3. Comparisons show that the NAPPO model is more accurate than the current procedures used to forecast officer basic pay.

RECOMMENDATION

It is recommended that the model be used by Navy management to provide future basic pay and mean LOS estimates.

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